

Benha University Faculty of Engineering- Shoubra Eng. Math. and Phy. Department		2 <sup>nd</sup> Year- Mech. Eng.(Power) Mathematics & Computers -A Date: 23 / 1 / 2011
الامتحان (5) أسئلة في صفحة واحدة و المطلوب الإجابة عن كل الأسئلة الزمن 3 ساعات		Marks
<p>[1] Solve the following P.D.E:</p> <p>(a) <math>u_x - 2u_y + u = 0</math></p> <p>(b) <math>u_{xx} - 3u_{xy} + 2u_{yy} = \cos(x + y)</math></p> <p>(c) <math>u_{xx} - 2u_{xy} + u_{yy} = x^2y^2</math></p> <p>(d) <math>u_{xx} - 4u_{xy} + 4u_{yy} - u_x + 4u = 0</math></p>	16	
<p>[2](a) Find <math>u, v</math> of <math>f(z) = z^2 + \ln(z + 1)</math> and show that they satisfy Riemann's equations.</p> <p>(b) Determine and sketch the image of the region <math>G: 1 \leq x \leq 2, 0 \leq y \leq \pi</math> under the function <math>f(z) = e^z</math>.</p> <p>(c) Write Maclurin's series of <math>f(z) = \ln(z + 1)</math> and find the sum:</p> $\cos \theta - \frac{1}{2} \cos 2\theta + \frac{1}{3} \cos 3\theta \dots$	5	5
<p>[3](a) Find the integrals: (i) <math>\oint_C \frac{e^z}{(z-\pi i)^2} dz</math>      (ii) <math>\oint_C \frac{\sin z}{z-2} dz</math> where <math>C</math> is <math> z - 2i  = 2</math></p> <p>(b) By Residue theorem, compute the integrals:</p> <p>(i) <math>\int_{-\infty}^{\infty} \frac{1}{(x^2-4x+5)^2} dx</math></p> <p>(ii) <math>\int_0^{2\pi} \frac{1}{5-3\sin\theta} d\theta</math></p>	6	10
<p>[4](a) Write the Fourier series of the function <math>f(x) = x, x \in [-\pi, \pi]</math>, <math>f(x + 2\pi) = f(x)</math></p> <p>Also, find the sum <math>1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots</math></p> <p>(b) Find the Fourier cosine series of the function <math>f(x) = x, x \in [0, \pi]</math>, <math>f(x + 2\pi) = f(x)</math></p> <p>Also, compute <math>1 + \frac{1}{3^4} + \frac{1}{5^4} + \dots</math> and <math>1 + \frac{1}{2^4} + \frac{1}{3^4} + \dots</math></p>	5	7
<p>[5](a) Write the Fourier sine integral of the function <math>f(x) = \begin{cases} x, &amp; 0 \leq x \leq 1 \\ 0, &amp; x &gt; 1 \end{cases}</math></p> <p>(b) Find the Fourier integral of the function <math>f(x) = \begin{cases} 2, &amp;  x  \leq 2 \\ 0, &amp;  x  &gt; 2 \end{cases}</math></p> <p>Also, find the integrals: <math>\int_0^{\infty} \frac{\sin t}{t} dt</math> and <math>\int_0^{\infty} \frac{\sin^2 y}{y^2} dy</math></p>	5	6